ZIMBABWE SCHOOL EXAMINATIONS COUNCIL (ZIMSEC)

ADVANCED LEVEL SYLLABUS

Mathematics
9164

EXAMINATION SYLLABUS FOR 2013 - 2017

Additional copies of the syllabus and specimen questions paper booklets can be ordered from ZIMSEC.

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MATHEMATICS 9164

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MATHEMATICS (9164)

GCE ADVANCED LEVEL

Introduction

In developing the scheme, attention was paid to the following considerations:

(i) the need to produce a Mathematics syllabus which provides continuity from O-Level ZGCE, through to the tertiary education;

(ii) the desire to produce examination papers which will enable candidates to demonstrate positive evidence of their attainment, and which at the same time will eliminate any adverse effects of question choice;

(iii) the desire to preserve those topics from the Mathematics syllabus (9202) which have proved to be of value and which are likely in future to be of value;

(iv) The desire to allow centres to choose from three different routes to A-Level Mathematics, depending on the choice of Pure Mathematics and/or Mechanics or Statistics or both in the broad area of 'applications';

(v) the desire to expose all candidates to some applications in both fields, Mechanics and Statistics.

Syllabus Aims

The syllabus is intended to provide a framework for 'A' Level courses that will enable students to:

(a) develop further the understanding of mathematical concepts and processes in a way that encourages confidence and enjoyment;

(b) develop a positive attitude to learning and applying Mathematics;

(c) acquire and become familiar with appropriate mathematical skills and techniques;

(d) appreciate mathematics as a logical and coherent subject;

(e) develop their ability to think clearly, work carefully and communicate mathematical ideas successfully;

(f) develop their ability to formulate problems mathematically, interpret a mathematical solution in the context of the original problem, and understand the limitations of mathematical models;

(g) appreciate how mathematical ideas can be applied in the everyday world;

(h) acquire a suitable foundation for the study of mathematics and related disciplines.

Assessment Objectives

The assessment will test candidates' abilities to:

(a) recall, select and use their knowledge of appropriate Mathematical facts, concepts and techniques in variety of context;

(b) construct rigorous mathematical arguments through appropriate use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions;

(c) evaluate mathematical models, including an appreciation of the assumptions made, and interpret, justify and present the results from a mathematical analysis in a form relevant to the original problem.

It is expected that Assessment Objectives (a) and (b) will apply to all components and that Assessment Objective (c) will apply mainly to Papers 2, 3 and 4 of mathematics.

In additions to the above Objectives, other objectives of more specialised relevance are listed at the start of the appropriate sections in the list of curriculum objectives.

Scheme of Assessment.

All papers will contain questions of various lengths with no restriction on the number of questions which may be attempted. On each paper, the total of the question marks will be 120. The length of each paper will be such that less able candidates will be able to demonstrate positive achievement. Questions in each section will appear in ascending order of their mark allocations and candidates are advised to attempt them sequentially. Candidates should be aware that credit for later parts of a question may generally be available even when earlier parts have not been completed successfully.

The examination will consist of two, equally weighted, three-hour papers. **Candidates will take Paper 1 and one of Papers 2, 3, 4.**

Paper 1 and 2

80% of the marks available in the examination will be allocated to questions on Pure Mathematics;

20% of the marks available in the examination will be allocated to questions on Applications.

Paper 1 and 3 or Paper 1 and 4
50% of the marks available in the examination will be allocated to questions on Pure Mathematics;

50% of the marks available in the examination will be allocated to questions on Applications.

PAPER	TOPICS	WEIGHT	MARKS	DURATION
Paper 1	Pure Mathematics 1-17	100%	120	3 hours
Paper 2	Pure Mathematics 1-21 Mechanics 1-4 Statistics 1-5	60% (72) 20% (24) 20% (24)	120	3 hours
Paper 3	Statistics 1-5 Mechanics 1-12	20% (24) 80 (96)	120	3 hours
Paper 4	Statistics 1-11 Mechanics 1-4	80% (96) 20% (24)	120	3 hours

Paper 1 Pure Mathematics – a paper containing about 17 questions set on topics 1-17 of the Pure Mathematics list (120 marks)

and one of the following three papers

Paper 2 Pure Mathematics, Mechanics and Statistics – a paper containing

- About 8 questions set on topics 1 – 21 of the Pure Mathematics list (72 marks)

- About 4 question (which will also be common to Paper 3 and 4 set on topics 1-4 of the mechanics list (24 marks)

- And about 4 questions (which will also be common to Paper 3 and Paper 4) set on topics 1-5 of the Statistics list (24 marks)

Paper 3 — Mechanics and Statistics – a paper containing about 12 questions set on topics 1-12 of the Mechanics List (96 marks)

- and about 4 questions, set on topics 1-5 of the Statistics List (24 marks)

Paper 4 — Statistics and Mechanics – a paper containing about 12 questions on topics 1-11 of Statistics List (96 marks)

- and about 4 questions set on topics 1-4 of the Mechanics List (24 marks)

SPECIFICATION GRID

PM = Pure Mathematics
M = Mechanics
S = Statistics

Component Skills	**Paper 1**	**Paper 2**	**Paper 3**	**Paper 4**
Skill 1 Knowledge Comprehension	PM = 50% 50%	PM = 18% M = 6% S = 6% 30%	M = 24% S = 6% 30%	S = 24% M = 6% 30%
Skill 2 Application Analysis	PM = 40% 40%	PM = 33% M = 11% S = 11% 55%	M = 44% S = 11% 55%	S = 44% M = 11% 55
Skill3 Synthesis Evaluation	PM = 10% 10%	PM = 9% M = 3% S = 3% 15%	M = 12% S = 3% 15%	S = 12% M = 3% 15%
TOTAL	**100%**	**100%**	**100%**	**100%**

Summary of Content for each paper

Detailed lists appear on pages 9 – 33

Paper 1 Pure Mathematics 3 hours (120 marks)

1 Indices and proportionality
2 Polynomials
3 Identities, equations and inequalities
4 The modulus function
5 Graphs and coordinate geometry in two dimensions
6 Vectors (1)
7 Functions
8 Sequences and series
9 Series expansions
10 Plane trigonometry
11 Trigonometrical functions
12 Logarithmic and exponential functions
13 Differentiation
14 Integration
15 First order differential equations
16 Numerical methods
17 Complex Numbers (1)

Paper 2 *Pure Mathematics, Mechanics and Statistics* *3 hours (120 marks)*

Pure Mathematics 72 Marks

1 – 17 As in Paper 1 above
18 Complex Numbers (11)
19 Vectors (11)
20 Mathematical Induction
21 Matrices

Mechanics 24 Marks

1 Forces and equilibrium
2 Kinematics of motion in a straight line
3 Newton’s laws of motion
4 Motion of a projectile

Statistics 24 Marks

1 Representation of data
2 Probability
3 Discrete random variables
4 Continuous distributions

Paper 3 Mechanics and Statistics 3 hours (120)

Mechanics 96 marks

1 Forces and equilibrium
2 Kinematics of motion in a straight line
3 Newton's laws of motion
4 Motion of a projectile
5 Momentum
6 Equilibrium of a rigid body under coplanar forces
7 Centre of mass
8 Hooke/s law
9 Energy, work and power
10 Uniform motion in a horizontal circle
11 Linear motion under a variable force
12 Simple harmonic motion

Statistics 24 marks

Topics 1-----5 as paper 2

Paper 4 Statistics and Mechanics 3 hours (120)

Statistics 96 marks

1 Representation of data
2 Probability
3 Discrete random variables
4 Continuous distributions
5 The Normal distribution
6 Linear combinations of random variables
7 Samples
8 Statistical inference
9 Tests
10 Bivariate data (Regression AND correlation)
11 The Poisson distribution

Mechanics 24 marks

Topics 1-----4 as Paper 2

CURRICULUM OBJECTIVES

The following pages contain detailed lists of curriculum objectives for each of the three broad areas.

Pure Mathematics
Mechanics
Statistics

It should be noted that individual questions may involve ideas from more than one section of the following list and that topics may be tested in the context of solving problems and in the application of Mathematics. In particular, candidates will be expected to develop understanding of the process of mathematical modelling through the study of one or more application areas.

The following skills will be needed:

- Abstraction from a real world situation to a mathematical description. The selection and use of a simple mathematical model to describe a real world situation.
- Approximation, simplification and solution.
- Interpretation and communication of mathematical results and their implications in real world.
- Progressive refinement of mathematical models.

Pure Mathematics List

THEME OR TOPIC		CURRICULUM OBJECTIVES **Candidates should be able to:**
1.	**Indices and proportionality**	Understand rational indices (positive, negative and zero), and recall and use rules of indices in the course of algebraic applications including the notation $\sqrt{a} = a^{\frac{1}{2}}$ and simple properties such as $\frac{9}{\sqrt{3}} = 3\sqrt{3}$ *and* $\sqrt{12} = 2\sqrt{3}$; express general laws in symbolic form, and derive equations involving a constant of proportionality from statements concerning direct and inverse variation (including joint variation).
2.	**Polynomials**	carry out operations of addition, subtraction, multiplication and division of polynomials; factorise quadratic polynomials (real factors with rational coefficients only); use the factor theorem to find factors and to evaluate unknown coefficients.
3.	**Identities, equations and inequalities**	understand the distinction between identities and equations, and use identities to determine unknown coefficients in polynomials; use the process of completing the square for a quadratic polynomial and determine extreme values of a quadratic polynomial; solve a pair of simultaneous equations at least one of which is linear and at most one of which is quadratic; recall an appropriate form for expressing rational polynomials in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than $(ax + b)(cx + d)(ex + f)$, $(ax + b)(cx + d)^2$, $(ax + b)(x^2 + c^2)$ including cases in which the degree of the numerator does not exceed that of the denominator; use the discriminant of a quadratic polynomial f(x) to determine the number of real roots of the equation f(x)=0; recognise and solve equations in *x* which are quadratic in some function of x, e.g. $x^2 – 5x^2 + 4 = 0$; solve cubic equations, in cases where at least one rational root may be found, by means of the factor theorem.

THEME OR TOPIC		CURRICULUM OBJECTIVES **Candidates should be able to:**
4.	**The Absolute Value**	understand the meaning of $\lvert x \rvert$, and use relations such as $\lvert a \rvert = \lvert b \rvert \Leftrightarrow a^2 = b^2$ and $\lvert x - a \rvert < b \Leftrightarrow a - b < x > a + b$ solving equations and inequalities.
5.	**Graphs and coordinate geometry in two dimensions.**	use rectangular cartesian coordinates, and understand the relationship between a graph and an associated algebraic equation; demonstrate familiarity with the forms of the graphs of y = kx^n, where n is a positive or negative integer or n = $\frac{1}{2}$, and k is a constant, y=f(x), where f(x) is a quadratic or cubic polynomial in factorised form; use sketch-graphs to illustrate solutions of equations and inequalities; calculate the distance between two points given in coordinate form, the gradient of the line-segment joining them, and the coordinates of the mid-point; find the equation of a straight line given sufficient information (e.g. the coordinates of two points on it, or one point on it and its gradient); interpret and use equations of the form $ax + by + c = 0$ and $y = mx + c$, including knowledge of the relationships involving gradients of parallel or perpendicular lines; recognise when an equation can be reduced to liner form, and use this process in solving problems (e.g. $y + xa^2 + b$ when y is plotted against x^2); recognise the equation of a circle andidentify its centre and radius; understand the use of a pair of parametric equations to define a curve, and use a given parametric representation of a curve in simple cases.
6.	**Vectors (I)**	use rectangular Cartesian coordinates to locate points in three dimensions and use standard notations of vectors, *i.e* $\begin{pmatrix} x \\ y \\ z \end{pmatrix}, xi + yi + zk, \overrightarrow{AB}, a$;

THEME OR TOPIC	CURRICULUM OBJECTIVES **Candidates should be able to:**
	carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms;
	use unit vectors, position vectors and displacement vectors;
	recall the definitions of and calculate the magnitude of a vector and the scalar product of two vectors (in either two or three dimensions);
	use the scalar product to determine the angle between two directions and to solve problems concerning perpendicularity of vectors.
7. Functions	understand the terms function, domain, range (image set) one-one function, inverse and composition of functions;
	Illustrate in graphical terms the relation between a one-one function and its inverse;
	understand and use the relationships between the graphs of $y = f(x)$, $y = af(x)$, $y = f(x) + a$, $y = f(x+a)$, $y = f(ax)$, where a is a constant, express the transformations involved in terms of translation, reflections and sketches, and recognise simple compositions such as $y = af(x + b)$ where b is also a constant;
	understand the relationship between the graphs of $y = f(x)$ and $y = \lvert f(x) \rvert$
8. Sequences and series	understand the idea of a sequence of terms, and use definitions such as $U_n = n^2$ and relations such as $U_{n+1} = 2U_n$ to calculate successive terms;
	recognise that a sequence may exhibit periodicity, may oscillate, converge to a limit or diverge, and determine the behaviour of a sequence in simple cases;
	use $\sum$ notation;
	recognise arithmetic and geometric progressions;
	use the formulae for the n^{th} term and for the sum of the first n terms to solve problems involving arithmetic or geometrical progressions;

THEME OR TOPIC	CURRICULUM OBJECTIVES **Candidates should be able to:**
	recall the condition for convergence of geometric series, and use the formula for the sum to infinity of a convergent geometrical series.
9. Series expansions	use the expansions of $(a + b)^n$, where n is a positive integer, and of $(1 + x)^n$, where n is a rational number and $\lvert x \rvert < 1$ (finding a general term is not included); recognise and use the notations n! and $\binom{n}{r}$; use the first few terms of the Maclaurin series e^x, sin x, cos x, In $(1 + x)$; derive and use the first few terms of the Maclaurin series of simple functions, for example, e^x sin x, In $(3 + 2x)$ (derivation of a general term is not included).
10. Plane trigonometry	use the sine and cosine rules and the formula $\Delta = \frac{1}{2}$ ab sin C for the area of a triangle; understand the definition of a radian, and recall and use the relationship between degrees and radians; use the formulae s = rθ and A = $\frac{1}{2}$ $r^2\theta$ for the arc length and sector area of a circle; recall and use small angles approximations for sin x, cos x and tan x.
11. Trigonometrical functions	use the six trigonometrical functions for angles of any magnitude; recall and use the exact values of the sine, cosine and tangent of 30^o, 45^o, 60^o, e.g. cos $30^o = \frac{\sqrt{3}}{2}$, and their radian equivalence; use the notation $\sin^{-1}x$, $\tan^{-1}x$ a and $\cos^{-1}$ to denote the principal values of the inverse trigonometric relations; relate the periodicity and symmetries of the sine, cosine and tangent functions to the form of their graphs, and use the concepts of periodicity and/or symmetry in relation to these function and their inverses; use trigonometrical identities for the simplification and

THEME OR TOPIC	CURRICULUM OBJECTIVES **Candidates should be able to:**
	exact evaluation of expressions and select an identity or identities appropriate to the context, showing familiarity in particular with the use of $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$ *and* $\frac{\cos\theta}{\sin\theta} \equiv \cot\theta$, $\sin^2\theta + \cos^2\theta \equiv 1$ and equivalent statements, the expansions of sin ($A \pm B$), the formulae for sin 2*A*, cos 2A, the expression of a sinθ + b cosθ in the forms $R \sin(\theta \pm \alpha)$ and R cos$(\theta \pm \alpha)$;
	find all the solutions, within a specified interval of the equations sin(kx) = c, cos(kx) = c, tan (kx) = c, and of equations easily reducible to these forms.
12. Logarithmic and exponential functions	understand the relationship between logarithms and indices, and recall and use the laws of logarithms (excluding change of base);
	sketch graphs of simple logarithmic and exponential functions;
	understand exponential growth and decay;
	use logarithms to solve equations of the form $a^x = b$, and similar inequalities;
	use logarithms to transform a given relationship to linear form, and hence to determine unknown constants by considering the gradient and/or intercept.
13. Differentiation	understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords;
	understand the idea of a derived function, and use the notations *f'(x), f''(x) etc,* $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ *etc*;
	use the derivatives of x^n (for any rational n) In x, e^x, sin x, cosx,, tan x, together with constant multiples, sums, differences, products, quotients and composites;
	apply differentiation to gradients, tangents, normals, increasing and decreasing functions, rates of change;
	locate stationary points, and distinguish (by any method) between maximum and minimum points (students should know that not all stationary points are maxima or minima, but knowledge of conditions of inflexion is not included);

THEME OR TOPIC	CURRICULUM OBJECTIVES **Candidates should be able to:**
	understand and use the relation $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$; find and use the first derivative of a function which is defined implicitly or parametrically.
14. Integration	understand indefinite integration as the reverse process of differentiation, and integrate, for example, x^n, (including the case where n = -1) e^x, *cos x*, sec^2x together with sums, differences and constant multiples of these, expressions involving a linear substitution (*e.g. sin (ax + b)*, applications involving the use of a double angle formula (e.g. $\int cos^2\ xdx$);
	recognise an integrand of the form $\frac{kf'(x),}{f(x),}$ and integrate for example, $\frac{x}{x^2+1}$ or tan x;
	use a given substitution to simplify and evaluate either a definite or an indefinite integral (as noted above, candidates will be expected to integrate expressions involving only linear substitution at sight, but any other substitutions that may be required will be given);
	evaluate definite integrals (including e.g. $\int_0^1 x^{-\frac{1}{2}} dx$ *and* $\int_0^a e^{-x}\ dx$);
	use integration to find the area of a region bounded by a curve and lines parallel to the coordinate axes or between two curves, and simple cases of volumes of revolution about one of the axes;
	recognise when an integrand can usefully be regarded as a product, and use integration by parts to integrate, for example, *x sin 2x*, x^2e^x, *in x;*
	integrate rational functions, with denominators of the form *(ax + b) (cx + d)*, by means of decomposition into partial fractions.
15. First order differential equations	formulate a simple statement involving a rate of change as a differential equation, including the introduction if necessary of a constant of proportionality;
	find by integration a general form of solution for a differential equation in which the variables are separable;

THEME OR TOPIC		CURRICULUM OBJECTIVES **Candidates should be able to:**
		understand that the general solution of a differential equation is represented in graphical terms by a family of curves, and sketch typical members of a family in simple cases; use an initial condition to find a particular solution of a differential equation.
16.	**Numerical methods**	understand the distinction between absolute and relative errors in data which is not known (or stored) precisely; make estimates of the errors that can arise in calculations involving inexact data, including the use, when appropriate of $\partial y \approx \frac{dy}{dx}\,\partial x$ to locate approximately a root of an equation by means of graphical considerations and/ or searching for sign-change; understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation; understand how a given simple iterative formula of the form $X_{n+1} = F(X_n)$ relates to the equation being solved, and use a given iteration to determine a root to a prescribed degree of accuracy (conditions for convergence are not included) understand,in geometrical terms,the working of the Newton Raphson method ,and derive and use iterations based on this method; understand that an iterative method may fail to converge to the required root; understand how the area under a curve may be approximated by areas of rectangles and/or trapezia, and use rectangles and/or trapezia to estimate or set bounds for the area under a curve (including the use of the trapezium rule).
17.	**Complex numbers (I)**	understand the idea of a complex number, recall the meaning of the terms ‘real part’, ‘imaginary part’ modulus;, argument’, “conjugate’, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal; carry out operations of addition, subtraction, multiplication and division of two numbers expressed in

THEME OR TOPIC		CURRICULUM OBJECTIVES Candidates should be able to:
		cartesian form $(x + iy)$;
		represent complex numbers geometrically by means of an Argand diagram; understand in simple terms the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying and dividing two complex numbers.
18.	**Complex numbers (II)**	use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs;
		carry out operations of multiplication and division of two complex numbers expressed in polar form $(r(\cos\theta + i\sin\theta) = re^{i\theta})$;
		illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram (e.g $\|z-a\| < k$, $\|z-a\|=\|z-b\|$, $arg(z-a) = a$);
		understand de Moivre's Theorem, for positive integer exponent, in terms of the geometrical effect of multiplication of complex numbers;
		use deMoivre's Theorem to express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle;
		find and use the nth roots of unity, e.g. to solve an equation of the form $z^n = (a + ib)$.
19.	**Vectors (2)**	understand the significance of all the symbols used when the equation of a straight line is expressed in either of the forms r = a + tb or $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$;
		understand the significance of all the symbols used when the equation of a plane is expressed in any of the forms $ax + by + cz = d$ or $(\mathbf{r} - \mathbf{a}).\ n = 0$ or $r = a + \lambda b + \mu c$;
		use equations of lines and planes to solve problems concerning distances, angles and intersections, and in particular: – determine whether two liens are parallel, intersect or are skew, and find the point of intersection of two lines when it exists, – determine whether a line lies in a plane, is parallel

THEME OR TOPIC		CURRICULUM OBJECTIVES Candidates should be able to:
		to a plane, or intersects a plane, and find the point of intersection of a line and a plane when it exists, – find the angle between two lines, the angle between a line and a plane, and the angle between two planes.
20.	**Mathematical induction**	use the method of mathematical induction to establish a given result (not restricted to summation series); recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy and carry this out in simple cases, e.g. to find the n^{th} derivative of xe^x.
21.	**Matrices**	carry out operations of matrix addition, subtraction and multiplication, and recognise the terms null (or zero) matrix and identity (or unit) matrix; recall the meaning of the terms 'singular' and non-singular' as applied to square matrices, and, for 2 x 2 and 3 x 3 matrices, evaluate determinants and find inverses of non-singular matrices; understand and use the result, for non-singular matrices, and $(AB)^{-1} = B^{-1} A^{-1}$; understand the use of 2 x 2 matrices to represent certain geometrical transformations in the x-y plane, and in particular: – recognise that the matrix product AB represents the transformation that results from the transformation represented by B followed by the transformation represented by A, – recall how the area scale-factor of a transformation is related to the determinant of the corresponding matrix, – find the matrix that represents a given transformation or sequence of transformations (understanding of the terms 'rotation', 'reflection', 'enlargement', 'stretch' and 'shear' will be required); formulate a problem involving the solution of 2 linear

THEME OR TOPIC	CURRICULUM OBJECTIVES **Candidates should be able to:**
	simultaneous equations in 2 unknowns (or 3 equations in 3 unknowns) as a problem involving the solution of a matrix equation, and vice versa.

Mechanics List

In addition to testing the assessment objectives listed on page 3 the assessment of the following section will test candidates' abilities to

- select the appropriate mechanical principles to apply in a given situation;
- understand the assumptions or simplifications which have to be made in order to apply the mechanical principles and comment upon them, in particular, the modelling of a body as a particle;
- use appropriate units throughout.

THEME OR TOPIC		CURRICULUM OBJECTIVES Candidates should be able to:
1.	**Forces and equilibrium**	identify the forces acting in a given situation;
		show understanding of the representation of forces by vectors, and find and use the resultants and components;
		understand and use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero and equivalently that the sum of the components in any given direction is zero, and the converse of this;
		recall that a contact force between two surfaces can be represented by two components, the 'normal force' and the 'frictional force';
		use the model of a 'smooth' contact and understand the limitations of the model;
		understand the concept of limiting friction and limiting equilibrium;
		recall the definition of the coefficient of friction, and use the relationship $F \leq \mu R$ or $F = \mu R$ as appropriate;
		recall and use Newton's third law.
2.	**Kinematics of motion in a straight line**	understand the concepts of distance and speed, as scalar quantities, and of displacement, velocity and acceleration, as vector quantities, and understand the relations between them;
		sketch and interpret (t, x) and (t, v) graphs in particular understand and use the facts that:
		– the area under (t,v) represents distance travelled,

THEME OR TOPIC	CURRICULUM OBJECTIVES Candidates should be able to:
	– the gradient of a (t,x) graph represents the velocity, – the gradient of a (t,v) graph represents the acceleration, – use appropriate formulae for motion with constant acceleration in a straight line.
3. Newton's laws of motion	recall and understand Newton's first and second laws of motion; apply Newton's laws to the linear motion of a body of constant mass moving under the action of constant forces (including friction); recall and understand the relationship between mass and weight; model, in suitable circumstances, the motion of a body moving vertically or on an inclined plane, as motion with constant acceleration and understand any limitations of this model; solve simple cases of the motion of two particles, connected by a light inextensible string which may pass over a fixed, smooth, light pulley or peg.
4. Motion of a projectile	model the motion of a projectile as a particle moving with constant acceleration and understand any limitations of this model; use horizontal and vertical equations of motion to solve problems on the motion of projectiles (including finding the magnitude and direction of the velocity at a given time or position and finding the range on a horizontal plane); derive and use the Cartesian equation of the trajectory of a projectile (including cases where the initial speed and/or angle of projection is unknown).
5. Momentum	recall and use the definition of linear momentum and show understanding of its vector nature; understand and use conservation of linear momentum in simple applications involving the direct collision of two bodies moving in the same straight line before and after impact, including the case where the bodies coalesce. Knowledge of impulse and of the coefficient of restitution is required.

THEME OR TOPIC		CURRICULUM OBJECTIVES **Candidates should be able to:**
6.	**Equilibrium of a rigid body under coplanar forces**	calculate the moment of a force about a point in 2 dimensional situations only (understanding of the vector nature of moments is not required); recall that if a body in equilibrium under the action of coplanar forces then the vector sum of the forces is zero and the sum of the moments of the forces about any point is zero, and the converse of this; solve problems involving the equilibrium of a single rigid body under the action of coplanar forces (problems set will not involve complicated trigonometry).
7.	**Centre of mass**	understand and use the result that the effect of gravity on a rigid body is equivalent to a single force acting at the centre of mass of the body; recall the position of the centre of mass of a uniform straight rod or circular hoop, of a uniform lamina in the shape of a rectangle or variable or a circular disc, and of a uniform solid or hollow cylinder or sphere (no integration or summation will be required); solve problems such as those involving a body suspended from a point and the toppling or sliding of a body on an inclined plane.
8.	**Hooke's law**	recall and use Hooke's law as a model relating the force in an elastic string or spring to the extension or compression, and understand and use the term 'modulus of elasticity'.
9.	**Energy, work and power**	understand the concept of the work done by a force and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force (use of the scalar product is not required); understand the concepts of gravitational potential energy, elastic potential energy and kinetic energy, and recall and use appropriate formulae; understand and use the relationship between the change in energy of a system and the work done by the external forces; use appropriately the principle of conservation of energy;

THEME OR TOPIC	CURRICULUM OBJECTIVES **Candidates should be able to:**
	recall and use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion;
	solve problems involving, for example, the instantaneous acceleration of a car moving on a hill with resistance.
10. Uniform motion in a horizontal circle	understand the concept of angular speed for a particle moving in a circle with constant speed and recall and use the relation $v = r\omega$ (no proof required);
	understand that the acceleration of a particle moving in a circle with constant speed is directed towards the centre of the circle, and has magnitude r^2 *or* v^2/r (no proof required);
	use Newton's second law to solve problems which can be modelled as the motion of a particle moving in a horizontal circle with constant speed.
11. Linear motion under a variable force	recall and use $\frac{dx}{dt}$ for velocity, and $\frac{dv}{dt}$ or $\frac{vdv}{dx}$ for acceleration, as appropriate;
	solve problems which can be modelled by the linear motion of a particle moving under the action of a variable force by setting up and solving an appropriate differential equation (problems set will require only the solution of those types of differential equation which are specified in section 15 of the Pure Mathematics list for this syllabus).
12. Simple harmonic motion	recall a definition of SHM and understand the concepts of period and amplitude; use standard SHM formulae;
	set up the differential equation of motion in problem leading to SHM, recall and use appropriate forms of solution, and identify the period and amplitude of the motion.

Statistics List

In addition to the Assessment Objectives listed on page 3, the assessment of the following section will test candidates' abilities to:

- select an appropriate statistical technique to apply in a given situation;
- comment on and interpret statistical results.

THEME OR TOPIC		CURRICULUM OBJECTIVES Candidates should be able to:
1.	**Representation of data**	appreciate, in simple terms, the importance of collecting data by a method appropriate to the purpose for which the data is to be used, and understand that experimental or other data may be subject to errors or uncertainties; understand the reasons for organising and presenting data in tabular or diagrammatic form, and discuss advantages and/or disadvantages that particular representation may have; select a suitable way of presenting raw statistical data; extract from a table or statistical diagram salient features of the data, and express conclusions verbally; construct and interpret histograms, frequency polygons, stem and leaf diagrams, box and whisker plots and cumulative frequency graphs, bar graph, pie chart; calculate or estimate graphically (as appropriate) and interpret measures of central tendency (mean, median, mode) and variation (inter-quartile range, variance, standard deviation for both grouped and ungrouped data).
2.	**Probability**	use addition and multiplication of probabilities, as appropriate, in simple cases; understand the meaning of exclusive and independent probabilities in simple cases, e.g. situations that can be represented by means of a tree diagram (the use of formal rules and notation such as $P \cup B) = P(A) + \ (B) - \ P(A \cap B) and\ P(A \cap B) = P(B\|A)$ will be expected. Exclude Bayes theorem.
3.	**Discrete random variables**	understand the concept of a discrete random variable; construct a probability distribution table relating to a given situation and calculate E(X) and Var(X);

THEME OR TOPIC	CURRICULUM OBJECTIVES **Candidates should be able to:**
	recall and use formulae for probabilities for the Binomial and Geometric distributions and model given situations by one of these as appropriate (and also the notation X~B(n,p));
	recall and use (without proof) the expectations (means) and variances of these distributions.
4. Continuous distributions	understand and use the concept of a probability density function, and recall and use the properties of a density function (which may be defined ‘piecewise’);
	use the probability density function to calculate the mean, mode and variance of a distribution, and in general, and in simple cases, use the result $E(g(X) = \int_0^1 g(x)f(x)dx$, where f(x) is the probability density function of the continuous random variable *x*, and *g(X)* is a function of *X*;
	understand and use the relationship between the probability density function and the (cumulative) distribution function and use either to evaluate the median, quartiles and other percentiles;
	use a probability density function or a (cumulative) distribution function in the context of a model, including, for example, the use of a Uniform (continuous) distribution.
5. The Normal distribution	understand the concept of a continuous random variable with particular reference to the Normal distribution;
	use the Normal distribution tables and standardise a Normal variable;
	use the Normal distribution as a model, where appropriate, and solve problems concerning a variable *X*, where $X \sim N(\mu, \sigma^2)$, including: (i) finding the value of $P(X) > x)$ given the values of x_1, μ σ, (ii) finding a relationship between x_1, μ and σ given the value of $P(X) > x_1)$
	use the Normal distribution as an approximation to the Binomial distribution where appropriate (*n* large enough to ensure that $np>5$ and $nq>5$), and apply a continuity correction.

THEME OR TOPIC		CURRICULUM OBJECTIVES **Candidates should be able to:**
6.	**Linear combinations of random variables**	recall and use the results in the course of problem solving that, for either discrete or continuous random variables: *(i)* $E(aX + b) = aE(X) + b)$ and $Var(aX + b) = a^2Var(X)$, *(ii)* $E(aX + bY) = aE(X) + bE(Y)$, *(iii)* $Var(aX + bY) = a^2Var(X) + b^2Var(Y)$ for independent X and Y; recall and use the results that: (i) if X has a Normal distribution, then so does $aX + b$, (ii) if X and Y have independent Normal distributions, then $aX + bY$ has a Normal distribution, (iii) if X and Y have independent Poisson distributions, then $X + Y$ has a Poisson distribution.
7.	**Samples**	understand the distinction between sample and a population and appreciate the necessity for randomness in choosing samples; explain in simple terms why a given sampling method may be unsatisfactory (Basic knowledge of sampling and survey methods is required); recognise that the sample mean can be regarded as a random variable and recall and use the fact that $E(\bar{X}) = \mu$ and that $Var(\bar{X}) = \frac{\sigma^2}{n}$ use the fact that $\bar{X}$ is Normal if X is Normal; use (without proof) the Central Limit Theorem where appropriate; calculate unbiased estimates of the population mean and variance from a sample (only a simple understanding of the term ‘unbiased ‘is required); determine, from a sample from a Normal distribution of known variance, or from a large sample, a confidence interval for the population mean.

THEME OR TOPIC		CURRICULUM OBJECTIVES **Candidates should be able to:**
8.	**Statistical inference**	understand and use the concepts of hypothesis (null and alternative), test statistic, significance level and hypothesis test (1-tail and 2-tail); formulate hypotheses and apply a hypothesis test in the context of a single observation from a population which has a Binomial distribution using either the Binomial distribution or the Normal approximation to the Binomial distribution; formulate hypotheses and apply a hypothesis test concerning population mean using: (i) a sample drawn from a Normal distribution of known variance using the Normal distribution, (ii) a small sample drawn from a Normal distribution of unknown variance using a t-test, (iii) a large sample drawn from any distribution of unknown variance using the Central Limit Theorem.
9.	**χ^2 tests**	fit a theoretical distribution, as prescribed by a given hypothesis, to a given data (questions will not involve lengthy calculations); use a χ^2 – test with the appropriate number of degrees of freedom to carry out the corresponding goodness of fit analysis (classes should be combined so that each expected frequency is at least 5); use a χ^2 – test with the appropriate number of degrees of freedom for independence in a contingency tale (Yates' correction is not required but classes should be combined so that the expected frequency in each cell is at least 5);
10.	**Bivariate data (Regression and correlation)**	understand the concepts of least squares, regression lines and correlation in the context of a scatter diagram; calculate, both from simple raw data and from summarised data, the equations of regression lines and the product moment correlation coefficient and appreciate the distinction between the regression line of y on x and that of x on y; select and use, in the context of a problem, the appropriate regression line to estimate a value, and understand the uncertainties associated with such estimations;

THEME OR TOPIC		CURRICULUM OBJECTIVES **Candidates should be able to:**
		relate, in simple terms, the value of the product moment correlation coefficient to the appearance of the scatter diagram with particular reference to the interpretation of cases when the value of the product moment correlation coefficient is close to 1, -1 or 0.
11.	**The Poisson distribution**	recall and use the formula for the probability that r events occur for a Poisson distribution with parameter μ (and also the notation $X \sim Po(\mu)$;
		recall and use the mean and variance of a Poisson distribution with parameter μ;
		understand the relevance of the Poisson distribution to the distribution of random events and use Poisson distribution as a model;
		use the Poisson distribution as an approximation to the Binomial distribution, where appropriate (approximately $n > 50$ and $np < 5$).

MATHEMATICAL NOTATION

The list which follows summarizes the notation used in the Council's Mathematics examinations. Although primarily directed towards Advanced Level, the list also applies, where relevant, to examinations at all other levels.

Mathematics Notation

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, ...\}$	the set with elements x_1, x_2 ...
$\{x: ....\}$	the set of all x such that ...
n(A)	the number of elements in set A
Ø	the empty set
ξ	universal set
A'	the complement of the set A
$\mathbb{N}$	the set of positive integers and zero $\{0, 1, 2, 3, ...\}$
$\mathbb{Z}$	the set of integers, $\{1, \pm 1, \pm 2, \pm 3, ...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, ...\}$
Z_n	the set of integers modulo n, $\{0, 1, 2, ..., n - 1\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \,\square\, \mathbb{Q}: x > 0\}$
$\mathbb{Q}^+_0$	the set of positive rational numbers and zero, $\{x, \square, \mathbb{Q}: x \,\square\, 0\}$
R	the set of real numbers
R^+	the set of positive real numbers, $\{x \,\square\, \mathbb{Q}: x > 0\}$
R^+_0	the set of positive real numbers and zero, $\{x, \square\, \mathbb{Q}: x \,\square\, 0\}$
R^n	the real n tuples
C	the set of complex number
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\not\subset$	is not a subset of
$\not\subseteq$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a,b]$	the closed interval $\{x \,\square\, R: a \,\square\, x \,\square\, b\}$
$[a, b)$	the interval $\{x \,\square\, R: a \,\square\, x < b\}$
$(a, b]$	the interval $\{x \,\square\, R: a < x \,\square\, b\}$
(a, b)	the open interval $\{x \,\square\, R: a < x < b\}$

yRx	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

2. ***Miscellaneous Symbols***

=	is equal to
≠	is not equal to
≡	is identical to or is congruent to
≈	is approximately equal to
≅	is isomorphic to
∝	is proportional to
<; «	is less than; is much less than
;≻	is less than or equal to or is not greater than
>; »	is greater than ; is much greater than
<	is greater than or equal to or is not less than
∞	infinity

3. ***Operations***

$a + b$	a plus b
$a - b$	a minus b
a x b, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, ${}^{a}/_{b}$	a divided by b
$a : b$	the ration of a to b
$\sum_{I=1}^{I=N} a_i$	$a_1 + a_2 + \ldots + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \varepsilon$N(0! = 1)
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$ for n, r, ε, N. 0 □ r □ n $\frac{n(n-1)\ldots(n-r+1)}{r!}$. for n ε,Q,r, εN

4. *Functions*

f	function f
f(x)	the value of the function f at x
f: $A \rightarrow B$	is a function under which each element of set A has an imagine in set B
f: x $\rightarrow y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
g o f, gf	the composite function of f and g which is defined by (g o f) (x) for gf(x) = g(f(x))
$\lim_{x \to a} f(x)$	the limit of f(x) as x tends to a
Δx; δx	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the nth derivative of y with respect to x
$f'(x), f''(x), \ldots, f^{(n)}(x)$	the first, the second, ..., nth derivatives of f(x) with respect to x
$\int y\,dx$	indefinite integral of y with respect to x
$\int_a^b y\,dx$	definite integral of y with respect to x for values of x between a and b
$\frac{\partial y}{\partial x}$	the partial derivative of y with respect to x
$\dot{x}, \ddot{x}, \ldots$	the first, second, ..., derivatives of x with respect to time.

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, exp x	exponential functions of x
$\log_a x$	logarithm to the base a of x
ln x	natural logarithm of x
lg x	logarithm of x to base 10

6. *Circular and Hyperbolic Functions and Relations*

sin, cos, tan cosec, sec, cot	}	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	}	the inverse circular relations
sinh, cosh, tanh cosech, sech, coth	}	the hypeolic functions
$\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$ cosech^{-1}, sech^{-1}, $\coth^{-1}$	}	the inverse hyperbolic relations

7. *Complex Numbers*

i	square root of -1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta)$, r □ $\mathbb{R}^+_0$ $= r\mathrm{e}^{\mathrm{i}0}$, r□ $\mathbb{R}^+_0$
Re z	the real part of z, Re $(x + \mathrm{i}y) = x$
Im z	the imaginary part of z, Im $(x + \mathrm{i}y) = y$
$\lvert z\rvert$	the modulus of z $\lvert x + \mathrm{i}y\rvert =$ □ $(x^2 + y^2)$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta)\rvert = r$
arg z	the argument of z, arg(r(cos θ + i sin θ)) -θ, $-\pi < \theta$ □ π
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

M	a matrix **M**
$\mathbf{M}^{-1}$	the inverse of the square matrix **M**
$\mathbf{M}^{\mathbf{T}}$	the transpose of the matrix **M**
det **M**	the determinant of the square matrix **M**

9. *Vectors*

a	the vector **a**
□ *AB*	the vector represented in magnitude and direction by the directed line segment *AB*
â	a unit vector in the direction of the vector **a**
i, j, k	unit vectors in the directions of the Cartesian coordinate axes

|**a**| the magnitude of **a**

$|AB|$ the magnitude of AB

a.b the scalar product of **a** and **b**

a x b the vector product of **a** and **b**

10. *Probability and Statistics*

A, B, C, etc	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
P (A)	probability of the event A
A'	complement of the event A, the event `not A'
P $(A\|B)$	probability of the events A given the event B
X, Y, R, etc	random variables
x, y, r, etc	values of the random variables X, Y, R, etc
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
p (x)	the value of the probability function $P(X = x)$
$p_1, p_2 \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
F(x), G(x), ...	the value of the (cumulative) distribution function P $(X \le x)$
E(X)	expectation of the random variable X
E[g(X)]	expectation of g (X)
Var (X)	variance of the random variable X
G(t)	the vale f the probability generation function for a random variable which takes integer values
B (n, p)	binomial distribution, parameters n and p
$N(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population means
σ^2	population variance
σ	population standard deviation
x	sample means
s^2	unbiased estimate of population variance from a sample, $s^2 = \frac{\Sigma(x - x)^2}{n - 1}$
φ	probability density function of the standardised normal variable with distribution N (0,1)
Φ	corresponding cumulative distribution function

p	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample
$\mathrm{Cov}\,(X, Y)$	covariance of X and Y

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Pure Mathematics

Backhouse, Houldsworth, Horill & Wood *Pure Mathematics* (Longman) 0582066581

Bostock & Chandler *Core Maths for A Level (*Second Edition) Stanley Thornes) 074871779

MEW Group *Exploring Pure Maths* (Hodder & Sroughton) 0340 53159 2

Perkins & Perkins *Advanced Mathematics – A Pure Course* (Collins) 000 322239 X

Sadler & Thorning *Understanding Pure Mathematics* (OUP) 019 914243 2

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