ZIMSEC

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ZIMBABWE SCHOOL EXAMINATIONS COUNCIL (ZIMSEC)

ADVANCED LEVEL SYLLABUS

Further Mathematics 9187

EXAMINATION SYLLABUS FOR 2013 - 2017

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FURTHER MATHEMATICS (9187)

This paper may not be taken in the same examination session as Paper 3 and 4 of the A-Level Mathematics syllabus (9164).

Syllabus Aims

See A-Level Mathematics syllabus

Assessment Objectives

See A-Level Mathematics syllabus

Scheme of Assessment

The marks in the examination will be allocated in equal proportions to Pure Mathematics and Applications. The examination will consist of two, equally weighted, 3-hour papers. Knowledge of the A-Level Mathematics syllabus for Paper 1 and 2 will be assumed.

Paper	**Type Of Paper And Topics**	**Marks**	**Duration**
1	Pure Mathematics 1 – 8	120	3 hours
2	Mechanics 1 – 9 Statistics 1 – 7	120	3 hours

<u>Paper 1 (120 marks)</u>
Pure Mathematics – this is a paper containing about 14 questions chosen from topics 1 – 8.

and

<u>Paper 2 (120 marks)</u>
Mechanical and Statistics – this is a paper containing about 14 questions chosen from topics 1 – 9 of the mechanics list and the topics 1 – 7 of the Statistics list.

<u>Specification Grid</u>

Component Skills	**Paper 1**	**Paper 2**
Skill 1 Knowledge Comprehension	Pure Mathematics = 55%	Mechanics = 18% Statistics = 18%
Skill 2 Application Analysis	Pure Mathematics = 40%	Mechanics = 18% Statistics = 18%
Skill 3 Synthesis Evaluation	Pure Mathematics = 5%	Mechanics = 14% Statistics = 14%

SUMMARY OF CONTENT FOR EACH PAPER

Paper 1 *Pure Mathematics* *3 Hours* *120 Marks*

1 Rational Functions
2 Summation of Series
3 Vectors (3)
4 Hyperbolic Functions
5 Differentiation And Integration
6 Differential Equations
7 Matrices
8 Groups

Paper 2 *Mechanics and Statistics* *3 Hours* *120 Marks*

1 Momentum
2 Equilibrium of a rigid body under coplanar forces
3 Centre of mass
4 Energy, work and power
5 Uniform motion in a horizontal circle
6 Linear motion under a variable force
7 Motion in a vertical circle
8 Simple harmonic motion
9 Hooke's Law

Statistics

1 The Poisson distribution
2 Samples
3 Statistical inference
4 X^2 tests
5 Bivariate data
6 Linear combinations of random variables
7 Probability generating functions

Curriculum Objectives

The curriculum objectives for each of the three broad areas, Pure Mathematics, Mechanics and Statistics are shown below.

It should be noted that topics may be tested in the context of solving problems and in the application of Mathematics, and that individual questions may involve ideas from more than one section of the following list. In addition, the curriculum objectives relating to Papers 1 and 2, of A-Level Mathematics syllabus are assumed.

PAPER 1 – PURE MATHEMATICS

Pure Mathematics List

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
1 Rational functions	– recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than $(ax + b)\ (cx + d)\ (ex + f)$ $(ax + b)\ (cx + d)^2$ $(ax + b)\ (x^2 + c^2)$ including cases in which the degrees of the numerator exceeds that of the denominator; – determine the salient features of the graph of a rational function including oblique asymptotes, for cases where the numerator and denominator are of degree at most 2.
2 Summation of series	– use of standard results for Σr, Σr^2 Σr^3, to find related sums; – use the method of differences to obtain the sum of a finite series, e.g. by expressing the general terms in partial fractions; – recognise, by direct consideration of a sum to n terms, when a series is convergent, and find the sum to infinity in such cases.
3 Vectors (3)	– recall the definition, in geometrical terms, of the vector product of two vectors, and, in cases where **a** and **b** are expressed in component form, calculate **a x b** in component form; – recall, the definition in geometrical terms of the vector triple scalar product of three vectors and in case where **a, b,** and **c** are expressed in component form calculate their triple scalar product.

THEME OR TOPIC	CURRICULUM OBJECTIVES *Candidates should be able to:*
	– use equations of lines and planes to solve problems concerning distances, angles and intersections and in particular find the line of intersection of two non-parallel planes; find the perpendicular distance from a point to a plane, and from a point to a line; find the shortest distance between two skew lines.
4 Hyperbolic functions	– recall definitions of the six hyperbolic functions in terms of exponentials, and sketch the graphs of simple hyperbolic functions; – derive and use identities such as $\cosh^2 x - \sinh^2 x \equiv 1$ and $\sinh 2x \equiv 2 \sinh x \cosh x$; – use the notations $\sinh^{-1}x$, $\cosh^{-1}x$; $\tanh^{-1}x$ to denote the principal values of the inverse hyperbolic relations, and derive and use expressions in terms of logarithms for these;
5 Differentiation and integration	– obtain an expression for $\frac{d^2y}{dx^2}$ in cases where the relation between y and x is defined implicitly or parametrically; – understand the relationship between the sign of $\frac{d^2y}{dx^2}$ and concavity, and locate points of inflexion as points at which $\frac{d^2y}{dx^2}$ changes signs; – derive and use the derivatives of $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, $\sinh x$, $\cosh x$, $\tanh x$, $\sinh^{-1}x$, $\cosh^{-1}x$, $\tanh^{-1}x$; – integrate $\frac{1}{\sqrt{(a^2 - x^2)}}$, $\frac{1}{(a^2 + x^2)}$, $\frac{1}{\sqrt{x^2 - a^2}}$, $\frac{1}{\sqrt{x^2 + a^2}}$, and use appropriate trigonometric or hyperbolic substitutions for the evaluation of definite or indefinite integrals;

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
	– integrate rational functions by means of decomposition into partial fractions; – derive and use reduction formulae for the evaluation of definite integrals in simple cases; – use integration to find the areas of the surface of revolution when an arc of a curve whose equation is expressed in cartesian coordinates or in terms of a parameter, is rotated about one of the coordinate axes.
6 Differential equations	– find the general solution of a first order linear differential equation by means of an integrating factor; – recall the meaning of the terms 'complementary function' and 'particular integral' in the context of linear differential equations, and recall that the general solution is the sum of the complementary functions and a particular integral; – find the complementary function of a first or a second order linear differential equation with constant coefficients; – recall the form of, and find, a particular integral for a first or second order linear differential equation in the case where $ax+b$ *or* ae^{bx} *or* $a\cos(px)$ + bsin(px) is suitable form, and in other cases find the appropriate coefficient(s) given a suitable form of particular integral; – use initial conditions to find a particular solution to a differential equation, and interpret the solution in the context of a problem modelled by a differential equation.

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
7 Matrices	– Understand the cases that may arise concerning the consistency or inconsistency of 2 or 3 simultaneous equations, interpret the cases geometrically in terms of lines or planes, relate them to the singularity or otherwise of the corresponding square matrix, and solve consistent systems.
8 Groups	– recall that a group consists of a set of elements, together with a binary operation which is closed and associative, for which an identity exists in the set, and for which every element has an inverse in the set;
	– use the basic group properties to show that a given structure is, or is not, a group (questions may be set on, for example, groups of matrices, transformations, integers modulo n);
	– use algebraic methods to establish properties in abstract groups in easy cases, e.g. to show that any group in which every element is self inverse is commutative;
	– recall the meaning of the term ‘order’, as applied both to groups and to elements of a group, and determine the order of elements in a given group;
	– understand the idea of a subgroup of a group, find subgroups in simple cases, and show that given subsets are, or are not, (proper) subgroups;
	– recall and apply LaGrange’s theorem concerning the order of a subgroup of a finite group (the proof of the theorem is not required);
	– recall the meaning of the term ‘cyclic’ as applied to groups, and show familiarity with the structure of finite groups up to order 7;

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
	• understand the idea of isomorphism between groups, and determine whether given finite groups are, or are not, isomorphic.

PAPER 2 – MECHANICS AND STATISTICS

Mechanics List

In addition to testing the Assessment Objectives listed in the A-Level Mathematics syllabus (Mechanic section), the assessment of the following section will test candidates' abilities to:

- select the appropriate mechanical principles to apply in a given situation
- understand the assumptions or simplifications which have to be made in order to apply the mechanical principles and comment upon them, in particular, the modelling of a body as a particle;
- use appropriate units throughout.

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
1 Momentum	– Recall and use the definition of linear momentum and show an understanding of its vector nature; – understand and use conservation of linear momentum in simple application involving the direct collision of two bodies moving in the same straight line before and after impact, including the case where the bodies coalesce (knowledge of impulse and of the coefficient of restitution is required).
2 Equilibrium of a rigid body under coplanar forces	– calculate the moment of a force about a point in 2 dimensional situations only (understanding of a vector nature of moments is not required); – recall that if a rigid body is in equilibrium under the action of coplanar forces then the vector sum of the forces is zero and the sum of the moments of the forces about any point is zero, and the converse of this; – solve problems involving the equilibrium of a single rigid body under the action of coplanar forces (problems set will not involve complicated trigonometry).
3 Centre of mass	– understand and use the result that the effect of gravity on the rigid body is equivalent to a single force acting at the centre of mass of the body;

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
	– recall the position of the centre of mass of a uniform straight rod or circular hoop, of a uniform lamina in the shape of a rectangle or a circular disc, and of a uniform solid or hollow cylinder or sphere (no integration or summation will be required); – solve problems such as those involving a body suspended from a point and the toppling or sliding of a body on an inclined plane.
4 Energy, work and power	– understand the concept of the work done by a constant force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force (use of the scalar product is required); – understand the concept of gravitational potential energy, elastic potential energy and kinetic energy, and recall and use appropriate formulae; – understand and use the relationship between the change in energy of a system and the work done by external forces; – use appropriately the principle of conservation of energy; – recall and use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of a motion; – solve problems involving, for example, the instantaneous acceleration of a car moving on a hill with resistance.
5 Uniform motion in a horizontal circle	– understand the concept of angular speed for a particle moving in a circle with constant speed, and recall and use the relation $v = r\omega$ (no proof required);

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
	– under that the acceleration of a particle in a circle with constant speed is directed towards the centre of the circle, and has magnitude rw^2 or v^2/r (no proof required); – use Newton's second law to solve problems which can be modelled as the motion of a particle moving in a horizontal circle with constant speed.
6 Linear motion under a variable force	– recall an use $\frac{dx}{dt}$ for velocity, and $\frac{dv}{dt}$ or $v\frac{dv}{dx}$ for acceleration, as appropriate; – solve problems which can be modelled by the linear motion of a particle moving under the action of a variable force, by setting up and solving an appropriate differential equation (problems set will require only the solution of those types of differential equations which are specified in the section of the Pure Mathematics list of the A-Level Mathematics syllabus).
7 Motion in a vertical circle	– recall and use the radical and transverse components of acceleration of a particle moving in a circle; – solve problems which can be modelled by the motion of a particle in a vertical circle without loss of energy (including finding the tension in a string or the normal contact force, and when these are zero, and conditions for complete circular motion).
8 Simple harmonic motion (SHM)	– recall a definition of SHM and understand the concepts of period and amplitude; use standard SHM formulae; – set up the differential equation of motion in problem leading to SHM, recall and use appropriate forms of solution, and identify the period and amplitude of the motion;

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
	– recognise situations where an exact equation of motion may be approximated by an SHM equation, carry out necessary approximations (e.g. small-angle approximations or binomial expansions) and appreciate the conditions necessary for such approximations to be useful (including the simple pendulum).
9 Hooke's Law	– recall and use Hooke's Law as a model relating to the force in an elastic string or spring to the extension or compression, and understand and use the term "modulus of elasticity".

Statistics List

In addition to the Assessment Objectives listed in the A-Level Mathematics syllabus (Statistics Section), the assessment of the following section will test candidates' abilities to:

- select an appropriate statistical technique to apply in a given situation.
- comment on and interpret statistical results.

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
1 The Poisson distribution	– recall and use the formula for the probability that r events occur for a Poisson distribution with parameter μ (and also the notation X ~Po(μ)); – recall and use the mean and variance of a Poisson distribution with parameter μ; – understand the relevance of the Poisson distribution to the distribution of random events and use the Poisson distribution as a model; – use the Poisson distribution as an approximation to the Binomial distribution, where appropriate (approximately $n > 50$ and $np < 5$).
2 Samples	– Understand the distinction between a sample and a population and appreciate the necessity for randomness in choosing samples;

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
	– explain in simple terms why a given sampling method may be unsatisfactory (a detailed knowledge of sampling and survey method is not required); – recognise that the sample can be regarded as a random variable and recall and use the fact that E $(\bar{X}) = \mu$ and that Var $(\bar{X}) = \frac{\sigma^2}{n}$; – use the fact that $(\bar{X})$ is Normal if X is Normal; – use (without proof) the Central Limit Theorem where appropriate; – calculate unbiased estimates of the population mean and variance from a sample (only a simple understanding of the term 'unbiased' is required); – determine, from a sample from a Normal distribution of known variance, or from a large sample, a confidence interval of the population mean.
3 Statistical inference	– understand and use the concepts of hypothesis (null and alternative), significance level, and hypothesis test (1-tail and 2-tail); – formulate hypotheses and apply a hypothesis test in the context of a single observation from a population which has a Binomial distribution, using either the Binomial distribution or the Normal approximation to the Binomial distribution; – formulate hypotheses and apply a hypothesis test concerning the population mean using: i. a sample drawn from a Normal distribution of known variance using the Normal distribution, ii. a small sample drawn from a Normal distribution of unknown variance using a *t*-test,

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
	iii. a large sample drawn from any distribution of unknown variance using the Central Limit Theorem.
4 X^2 tests	– fit a theoretical distribution, as prescribed by a given hypothesis, to given data (questions will not involve lengthy calculations); – use a x^2 test with the appropriate number of degrees of freedom to carry out the corresponding goodness of fit analysis (classes should be combined so that each expected frequency is at least 5) – use a x^2 test with the appropriate number of degrees of freedom for independence in a contingency table (Yates' correction is not required but classes should be combined so that the expected frequency in each cell is at least 5).
5 Bivariate data	– understand the concepts of least squares, regression lines and correlation in the context of a scatter diagram; – calculate, both from simple raw data and from summarised data, the equation of regression lines and the product moment correlation coefficient and appreciate the distinction between the regression line of *y* on *x* and that of *x* on *y*; – select and use, in the context of a problem, the appropriate regression line to estimate a value and understand the uncertainties associated with such estimations; – relate, in simple terms, the value of the product moment correlation coefficient to the appearance of the scatter diagrams with particular reference to the interpretation of cases when the value of the product moment correlation coefficient is close to 1, -1 or 0.

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
6 Linear combinations of random variable	– recall and use the results in the course of problem solving that, for either discrete or continuous random variables: (i) E $(aX + b) = aE(X) + b$ and $Var(aX + (b) = a^2Var(X)$, *(ii)* $E(aX + bY) = aE(X) + bE(Y)$, (iii) $Var(aX + bY) = a^2Var(X) + b^2Var(Y)$ for independence X and Y, – recall and use the results that: i. if X has a Normal distribution, then so does a$X + b$, ii. if X and Y have independent Normal distributions, then $aX + bY$ has a Normal distribution, iii. if X and Y have independent Poisson distributions, then $X + Y$ has a Poisson distribution.
7 Probability generating functions	– understand the concepts of a probability generating function and construct the probability generating function for probability distributions such as the Uniform (discrete), Binomial, Geometric and Poisson; – recall and use the formulae for the mean and variance of a random variable, in terms of its probability generation function, and use these formulae to calculate the mean and variance of probability distributions such as a the Uniform (discrete), Binomial, Geometric and Poisson; – use the result that the probability generating function of the sum of independent random variables is a product of the probability generating functions of those random variables.

MATHEMATICAL NOTATION

The list which follows summarizes the notation used in the Council's Mathematics examinations. Although primarily directed towards Advanced Level, the list also applies, where relevant, to examinations at all other levels.

Mathematics Notation

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, ...\}$	the set with elements x_1, x_2 ...
$\{x: ....\}$	the set of all x such that ...
n(A)	the number of elements in set A
Ø	the empty set
ξ	universal set
A'	the complement of the set A
$\mathbb{N}$	the set of positive integers and zero {0, 1, 2, 3, ...}
$\mathbb{Z}$	the set of integers, {1, ±1, ±2, ±3, ...}
$\mathbb{Z}^+$	the set of positive integers, {1, 2, 3, ...}
Zn	the set of integers modula n, {0, 1, 2, ..., n - 1}
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \notin \mathbb{Q}: x > 0\}$
$\mathbb{Q}^+$	the set of positive rational numbers and zero, $\{x, \notin, \mathbb{Q}: x \geq 0\}$
R	the set of real numbers
R^+	the set of positive real numbers, $\{x \notin \mathbb{Q}: x > 0\}$
R^+0	the set of positive real numbers and zero, $\{x, \notin \mathbb{Q}: x \geq 0\}$
R^n	the real n tuples
C	the set of complex number
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\subseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a,b]$	the closed interval $\{x \notin R: a \leq x \leq b\}$
$[a, b)$	the interval $\{x \notin R: a \leq x < b\}$
$(a, b]$	the interval $\{x \notin R: a < x \leq b\}$
(a, b)	the open interval $\{x \notin R: a < x < b\}$
yRx	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$; «	is less than; is much less than
$\leq$; $\succ$	is less than or equal to or is not greater than
$>$; »	is greater than ; is much greater than
$\geq$;_	is greater than or equal to or is not less than
∞	infinity

3. *Operations*

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, , a/b	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\|a\|$	the modulus of the real number a
$n!$	n factorial for $n \varepsilon N (0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$ for n, r, ε, N. $0 \le r \le n$
	$\frac{n(n-1)\,...\,(n-r+1)}{r!}$. for n ε,Q,r, εN

4. *Functions*

f	function f
f(x)	the value of the function f at x
f: $A \rightarrow B$	is a function under which each element of set A has an imagine in set B
f: x← y	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
g o f, gf	the composite function of f and g which is defined by (g o f) (x) for gf(x) = g(f(x))
$\lim_{x \to a} f(x)$	the limit of f(x) as x tends to a
Δx; δx	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the nth derivative of y with respect to x
f' (x), f''(x), ..., $f^{(n)}(x)$	the first, the second, ..., nth derivatives of f(x) with respect to x
_ydx	indefinite integral of y with respect to x
_$_a^b$ ydx	definite integral of y with respect to x for values of x between a and b
$\frac{\partial y}{\partial x}$	the partial derivative of y with respect to x
$\dot{x}, \ddot{x}, ...$	the first, second, ..., derivatives of x with respect to time.

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, exp x	exponential functions of x
$\log_a x$	logarithm to the base a of x
ln x	natural logarithm of x
lg x	logarithm of x to base 10

6. ***Circular and Hyperbolic Functions and Relations***

sin, cos, tan cosec, sec, cot	}	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	}	the inverse circular relations
sinh, cosh, tanh cosech, sech, coth	}	the hyperbolic functions
$\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$ cosech^{-1}, sech^{-1}, $\coth^{-1}$	}	the inverse hyperbolic relations

7. ***Complex Numbers***

i	square root of -1
z	a complex number, $z = x + iy$ $= r(\cos\theta + i\sin\theta),\ r \in R^+_0$ $= re^{i0},\ r \in R^+_0$
Re z	the real part of z, Re $(x + iy) = x$
Im z	the imaginary part of z, Im $(x + iy) = y$
$\lvert z\rvert$	the modulus of z $\lvert x + iy\rvert = \sqrt{(x^2 + y^2)}$, $\lvert r(\cos\theta + i\sin\theta)\rvert = r$
arg z	the argument of z, arg(r(cos θ + i sin θ)) -θ, $-\pi < \theta \le \pi$
z^*	the complex conjugate of z, $(x + iy)^* = x - iy$

8. ***Matrices***

M	a matrix **M**
$\mathbf{M}^{-1}$	the inverse of the square matrix **M**
$\mathbf{M}^{T}$	the transpose of the matrix **M**
det **M**	the determinant of the square matrix **M**

9. ***Vectors***

a	the vector **a**
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
â	a unit vector in the direction of the vector **a**
i, j, k	unit vectors in the directions of the Cartesian coordinate axes
|**a**|	the magnitude of **a**
$\lvert\overrightarrow{AB}\rvert$	the magnitude of $\overrightarrow{AB}$
a.b	the scalar product of **a** and **b**
a x b	the vector product of **a** and **b**

10. ***Probability and Statistics***

A, B, C, etc	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
P (A)	probability of the event A
A'	complement of the event A, the event `not A'
P ($A\|B$)	probability of the events A given the event B
χ, Y, R, etc	random variables
x, y, r, etc	values of the random variables X, Y, R, etc
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
p (x)	the value of the probability function $P(X = x)$
$p_1, p_2 \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
F(x), G(x), ...	the value of the (cumulative) distribution function P $(X \le x)$
E(χ)	expectation of the random variable X

(continued)

E[g(χ)]	expectation of g (X)
Var (χ)	variance of the random variable X
G(t)	the value of the probability generating function for a random variable which takes integer values
B (n, p)	binomial distribution, parameters n and p
N(μ, σ^2)	normal distribution, mean μ and variance σ^2
μ	population means
σ^2	population variance
σ	population standard deviation
x	sample means
s^2	unbiased estimate of population variance from a sample, $s^2 = \frac{\Sigma(x - x)^2}{n - 1}$
φ	probability density function of the standardised normal variable with distribution N (0,1)
Φ	corresponding cumulative distribution function
p	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample
Cov (χ, Y)	covariance of X and Y